EXPERIMENTAL INVESTIGATION OF FLOW IN A TRENCH

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An experimental investigation was made of the flow of a viscous incompressible liquid in a trench of square transverse cross section, using a laser Doppler velocimeter. The investigation was made with two values of the Reynolds number Re, corresponding to laminar and turbulent flow conditions in the channel. The experimental data show that a core with a constant vorticity is formed in the trench, that a jet propagates near the walls of the trench, and that there are secondary eddies in the corners of the trench. The motion of a viscous liquid in a trench of rectangular cross section is part of a broad class of breakaway flows. Experimental data on the investigation of flow in trenches are extremely few. A majority of the existing information is limited to visual observations [1-4]. In [2, 5, 6] the question of the unstable character of flow in trenches was discussed. Quantitative measurements of stable eddy flows in trenches were made in [7-9] using a thermoanemometer, and in [7] measurements were made of the pressure at the bottom and walls of trenches; there are data on the distribution of the velocity in the middle sections of trenches. In [8] the mean velocity, the intensity of the turbulence, and the stress of the turbulent flow were obtained in several sections parallel to the side walls of the trench. In [9] a measurement was made of the velocities also in two cross sections of a trench in which one component of the velocity prevails. A brief analysis of the existing experimental results shows that these data are insufficient to form a detailed representation of the character of flow in a trench.

1. Description of Experimental Unit and Method of Measurements

The present work was carried out in a hydrodynamic loop, in which a constant-head tank served to maintain the mass flow rate of the water. Water is fed by a pump from a lower tank to the upper tank (constant-head); it then enters a stilling chamber with a cross section of 200 × 200 mm, in which equalizing grids are installed, and then, through a diffuser, it enters a flat section with a cross section of 100 × 10 mm, consisting of three sections. The experimental trench, with dimensions of its cross section of 40 × 40 mm, is located at the lower wall of the third cross section of the section. The width of the trench is equal to the width of the flat section (100 mm). The side walls of the third section are made of optical glasses; they are installed flush with the side walls of the preconnection section. The trench is arranged at a distance of 1500 mm from the start of the flat section. After the experimental channel, the water passes through a regulating valve into the lower tank, in which a heat exchanger is installed.

The velocity of the motion of the liquid was measured using a laser Doppler velocimeter [10]. The Doppler frequency is connected with the parameters of the optical scheme and with the vector of the velocity by the relationship (with the condition $2\alpha \leq 15°$)

$$f_g = [(\overline{U} \cos \gamma \cdot \cos \varphi) b]/\lambda F,$$

where $\overline{U}$ is the vector of the velocity, arbitrarily oriented in space, m/sec; γ is the angle between the projection of the vector of the velocity on the planes of the incident beams and the normal to the bisectrix of the angle between the beams; φ is the angle of inclination of the vector of the velocity to the plane of the incident beams; 2α is the angle between the incident beams; λ is the wavelength of the radiation of the laser; b is the

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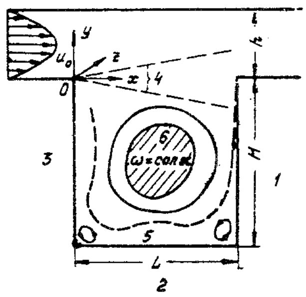

Fig. 1

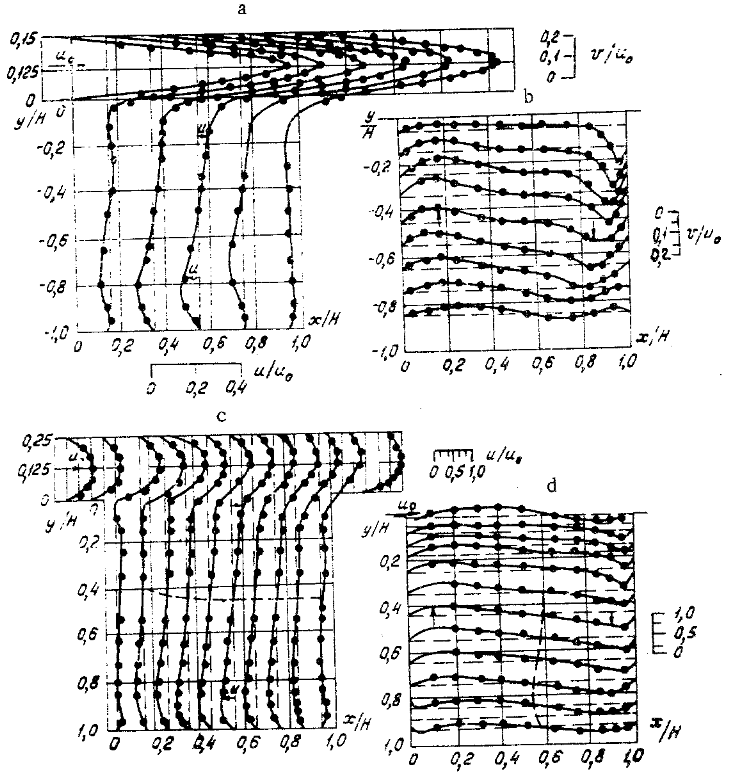

Fig. 2

distance between the centers of the forming diaphragms; F is the focal distance of the directing objective; f_g is the Doppler frequency, MHz.

In the present optical scheme, the coefficient of proportionality between the Doppler frequency and the measured component of the velocity was found equal to 11.5, i.e.

$$f_g = 11.5\ \bar{U}$$

$$(b = 4.40\ \text{mm},\ F = 80\ \text{mm},\ \gamma = F = 0,\ \lambda = 0.6328 \cdot 10^{-3}\ \text{mm}).$$

In this work, consecutive measurements were made of the transverse and longitudinal components of the mean velocity of the flow. With measurement of the transverse components of $\bar{U}$, the plane of the incident beams is arranged strictly parallel to the bottom of the trench, and the bisectrix of the angle between the incident beams is parallel to the ribs of the trench. The focal region is located at an equal distance from the side walls of the channel. The dimensions of the focal region are $100 \times 100 \times 800\ \mu$. The zero values of the coordinates were determined visually from the contact between the walls of the channel and the focal region. The width of the channel along the path of a beam is 100 mm, and the thickness of the optical glasses is 10 mm. At a distance of 50 mm from the focal region, the diameter of the laser beam is 0.5-0.6 mm. The minimal distance from the walls at which the velocity was measured was ~1 mm. The averaging time with the measurement of one value of the velocity was established within the limits 3-5 sec. In all the figures, the values of the velocity given are the average values of 3-4 measurements. After measurements of the longitudinal component of the vector of the velocity, the optical scheme of the laser Doppler velocimeter was changed over for measurement of the transverse component of the velocity.

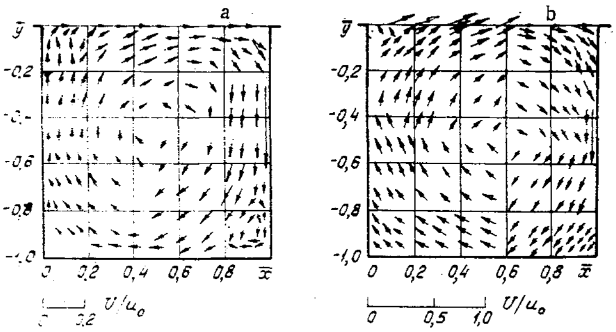

Fig. 3

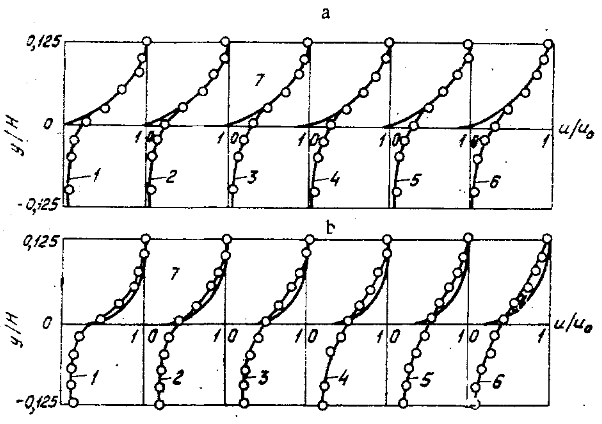

Fig. 4

The velocity was measured at 2-3 cross sections of the channel ahead of the trench, along the whole section of the trench with a spacing of 1-2 mm (both with respect to the longitudinal and the transverse coordinates), and in 1-2 cross sections in the channel beyond the trench.

With Re = 1500, there are zones with very small values of the mean velocity, going beyond the limit of the measurement by the electronic part of the laser Doppler velocimeter ($u < 1$ mm/sec). In this case, the velocity was measured using a correlator. The Doppler signal from the output of a differential amplifier entered the input of the correlator. On the display screen of the correlator there was obtained the curve of an autocorrelation function, having the form of a damped cosinusoid. Measuring the period of the autocorrelation function, the mean value of the period of the Doppler frequency was determined.

Investigations of flow in a trench were made with two values of the Reynolds number – Re = 1500 (laminar conditions) and Re = 15,000 (turbulent conditions):

$$\mathrm{Re} = u_0 2h/\nu,$$

where u_0 is the velocity of the liquid at the axis of the channel, in a cross section located 10 mm upstream from the trench; h is the height of the channel (Fig. 1; 1 is the rear wall of the trench; 2 is the bottom; 3 is the front wall; 4 is the mixing region; H is the height of the trench; L is the length of the trench).

2. Laminar Flow of a Liquid

For the case of the laminar flow of a liquid in a channel, Fig. 2a,b (v is the transverse component of the velocity) gives the distribution of the longitudinal and transverse components of the mean velocity of the flow over the whole investigated region. Figure 3a shows the field of the mean velocity in the trench. In the channel ahead of the trench the experimental profile of the velocity coincides with the theoretical Hagen–Poiseuille distribution of the velocity in the gap between two parallel plates:

$$u/u_0 = 4\,(y/h) - 4\,(y/h)^2.$$

From Fig. 2a,b and Fig. 3a, it can be seen that a fully developed circulating motion arises in the trench. The region adjacent to the upper section of the trench can be regarded as the zone of the mixing of the jet, forming after the breakaway of the flow at the point x = 0, y = 0 with the flow in the trench. After the impact of the jet against the rear wall of the trench, a jet is formed near the wall, propagating along all the walls of the trench. In the right-hand lower corner of the trench (see Fig. 3a) a secondary eddy is formed, rotating in the opposite direction (in distinction from the main eddy). The zone occupied by this eddy makes up 6% of the

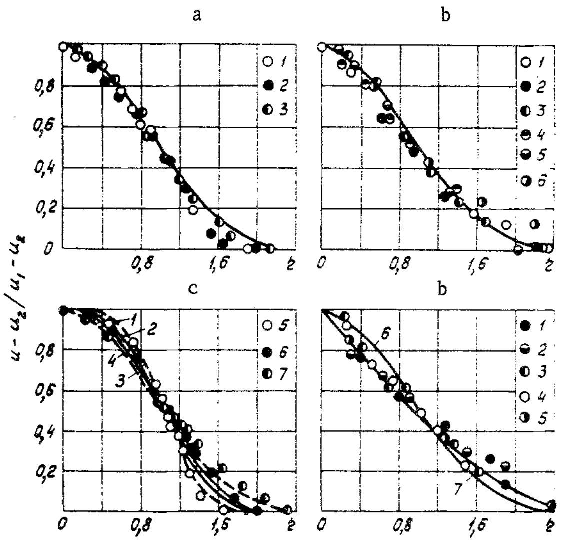

Fig. 5

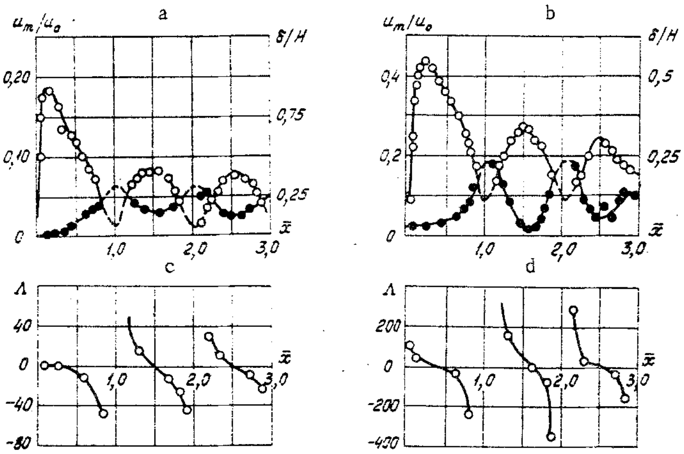

Fig. 6

area of the transverse cross section of the trench. In the left-hand lower corner a stagnant zone is formed. The area occupied by this zone is 2% of the area of the transverse cross section of the trench. Let us consider in more detail the flow in the different regions of the trench.

Mixing Zone. The profiles of the longitudinal component of the mean velocity in the region adjacent to one plane zOx (see Fig. 1) for several cross sections of the jet are shown in Fig. 4a ($1 - \bar{x} = x/H = 0.25$; $2 - \bar{x} = 0.35$; $3 - \bar{x} = 0.55$; $4 - \bar{x} = 0.65$; $5 - \bar{x} = 0.75$; $6 - \bar{x} = 0.95$). The value $\bar{y} = y/H = 0.125$ corresponds to the arrangement of the axial line of the channel. Here, for purposes of comparison, the parabolic profile 7 is also plotted. The experimental profiles of the velocity in all the cross sections of the jet, starting from $\bar{x} = 0.25$, practically coincide among themselves and, in the zone near the axis, also with the initial parabolic profile of the velocity in the channel.

A line, which is a prolongation of the axis of the channel above the trench, can be arbitrarily taken as the axial line of the jet, forming in the mixing zone after the breakaway of the jet at the point $x = 0$, $y = 0$. In Fig. 5a these same profiles of the velocity are represented in the coordinates $\bar{u}$, $\eta_{1/2}$, where $\bar{u} = (u - u_2)/(u_1 - u_2)$; u_1 is the velocity at the axis of the jet ($\bar{y} = 0.125$); u_2 is the velocity at the outer boundary of the jet $\eta_{1/2} = y/\delta_{1/2}$; $\delta_{1/2}$ is the arbitrary thickness of the jet (the distance from the axis of the jet, where $\bar{u} = 1$, to the point at which $\bar{u} = 0.5$) ($1 - \bar{x} = 0.15$; $2 - x = 0.45$; $3 - \bar{x} = 0.85$). As the outer boundary of the jet there is taken the point at which $\partial u/\partial y = (\partial u/\partial y)_{core} = \text{const}$. Here there is also plotted (solid line) the universal profile of the velocity in accordance with the formula [11]

$$(u-u_2)/(u_1-u_2) = 1 - 6\eta^2 + 8\eta^3 - 3\eta^4, \qquad (2.1)$$

where $\eta = y/\delta$; δ is the half-width of the jet. The experimental profiles of the velocity in the jet practically coincide with the universal profile of the velocity.

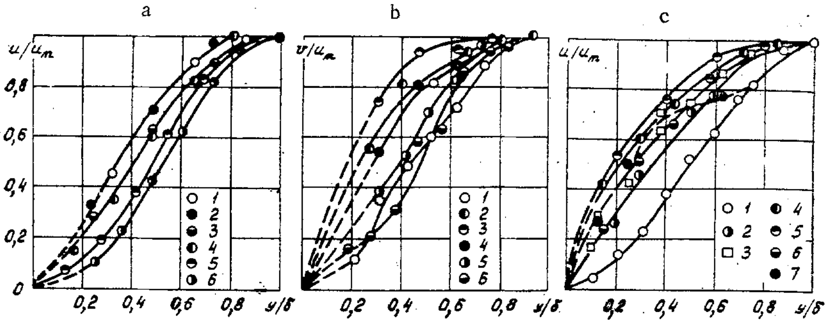

Fig. 7

The Boundary Layer. As has been shown above, a jet propagates near the walls of the trench. The jet near the wall is generally divided into two parts: a boundary layer at the wall and a jet region [11-14]. We assume that the boundary layer of liquid extends from the wall of the trench to the point at which the velocity has a maximum. We construct the distribution of the velocity at the limit of the boundary layer and the thickness of the boundary layer (in Fig. 6a denoted by the light and dark circles, respectively) along the longitudinal coordinate, arbitrarily extending the walls of the trench into planes. Knowing this dependence, we can construct the distribution of the shape parameter Λ, characterizing the effect of the longitudinal gradient of the velocity on the laminar boundary layer (Fig. 6c). The values of the shape parameter were determined using the formula

$$\lambda = (\delta^2/\nu)\cdot du_{max}/dx.$$

Figure 7 shows experimental profiles of the longitudinal component of the mean velocity in the boundary layer at the rear wall, the bottom, and the front wall of the trench. At the rear wall of the trench (Fig. 7a, $1 - \bar{x} = 0.35$; $2 - \bar{x} = 0.45$; $3 - \bar{x} = 0.5$; $4 - \bar{x} = 0.6$; $5 - \bar{x} = 0.7$; $6 - \bar{x} = 0.75$), the profile of the velocity, with a decrease in the shape parameter Λ from 0 to 40, was restructured from full to breakaway, which is in qualitative agreement with the results of a Pohlhausen calculation for a flat boundary layer in the presence of a pressure gradient. For an ordinary laminar boundary layer, the shape parameter Λ varies within the limits $-12 \leq \Lambda \leq 12$. In our case (see Fig. 6c), the breakaway of the boundary layer set in considerably later, which is connected with the presence of a negative vorticity at the outer limit of the boundary layer. With a transition from the rear wall to the bottom of the trench (Fig. 7b, $1 - \bar{x} = 1.25$; $2 - \bar{x} = 1.35$; $3 - \bar{x} = 1.45$; $4 - \bar{x} = 1.65$; $5 - \bar{x} = 1.8$; $6 - \bar{x} = 1.85$), the sign of the shape parameter changes jumpwise to the opposite, but the breakaway profile under these circumstances is transferred from the rear wall to the bottom of the trench, as a result of which the character of the distribution of the velocity over the cross section of the boundary layer does not correspond to the value and sign of the shape parameter. Further on, the flow developed in such a way that, with a decrease in Λ to zero, the profile of the velocity becomes full, and, with a further decrease in Λ ($\Lambda < 0$), it is restructured to a breakaway profile. An analogous character of the change in the velocity profile is also observed at the front wall of the trench (Fig. 7c, $1 - \bar{x} = 2.15$; $2 - \bar{x} = 2.2$; $3 - \bar{x} = 2.25$; $4 - \bar{x} = 2.3$; $5 - \bar{x} = 2.5$; $6 - \bar{x} = 2.75$; $7 - \bar{x} = 2.9$). This argues that, in the calculation of such a unique boundary layer, developing at the bottom and the front wall of the trench, it is impossible to use the methods of calculation for an ordinary laminar boundary layer in the presence of a pressure gradient. The profile of the velocity at the front wall of the trench has an anomalous character at the point of breakaway of the flow ($x = 0$, $y = 0$).

The Outer Part of the Jet near the Wall. By the outer part of the jet we shall understand the region of the flow lying between the core of the flow and the boundary layer. It is well known that [11, 14] in the outer (jet) part of the jet near the wall at the point of a maximal velocity the tangential stress is not equal to zero, and the line of maximal velocities is not the line of the flow. Direct measurements [15] show, however, that the shift of the point of zero tangential stress with respect to the point of maximal velocity is insignificant; therefore, it can be assumed that the vectors of the maximal velocities lie at the line of the flow. Let us consider cross sections of the jet perpendicular to the vectors of the maximal velocities. We find the value of the longitudinal component of the velocity by projecting the vector of the velocity at each point of the cross section on the direction of the vector of the maximal velocity. In Fig. 5b ($1 - \bar{x} = 0.20$; $2 - \bar{x} = 0.35$; $3 - \bar{x} = 0.50$; $4 - \bar{x} = 0.65$; $5 - \bar{x} = 0.95$, $\bar{x}$ is reckoned along the line of the flow), it can be seen that the profiles of the longitudinal component of the velocity in the outer part of the jet near the wall coincide with the universal profile of the velocity for a plane jet [11].

An analysis of the experimental data showed that the kinematic momentum of the part of the jet in the

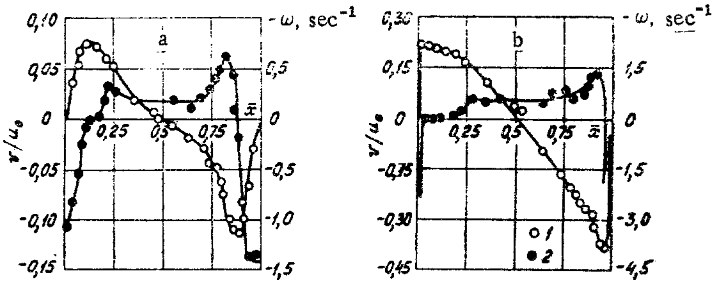

Fig. 8

zone of the shift (below the line $y = 0$) is less than the momentum of the jet in a cross section of the jet located immediately after the rotation of the jet with respect to the trench. The change of the maximal velocity in the jet near the wall along the line of the flow in the trench practically coincides with a dependence obtained for a flat free laminar jet [16]:

$$u_{max} = 0.454(K^2/\nu x)^{1/3},$$

where K is the kinematic momentum of the jet. The initial value of the longitudinal coordinate x_0 was determined using this same formula from measurements of the values of u_{max} and K in the initial cross section of the jet.

Core. Having the values of the longitudinal and transverse components of the velocity over the whole cross section of the trench, we can calculate the vortical stress ω:

$$\omega = 1/2\,[(\partial v/\partial x) - (\partial u/\partial y)],$$

where u is the component of the velocity directed along the x axis; v is the component of the velocity directed along the y axis.

Figure 8a [1) v/u_0; 2) ω] shows the distribution of the vorticity in a plane parallel to the bottom of the trench, arranged at a distance of H/2 from the bottom. At the center of the trench there is a zone with constant vorticity. In the outer part of the jet near the wall, propagating along the walls of the trench, there is an increase in the vorticity. This increase is most appreciable at the rear wall of the trench. In the region of the boundary layer the vorticity changes its sign to the opposite.

3. Turbulent Flow

Very different sets of flow conditions are possible in a trench. With laminar flow in the channel ahead of the trench, in the mixing zone there can be either laminar or turbulent flow. Let us evaluate the experimental results of an investigation of the flow of a liquid in a channel and in a trench with Re = 15,000. Under these circumstances, in the channel there are turbulent flow conditions with a velocity profile close to exponential ($n = 1/7$). Figure 2c,d gives the distribution of the longitudinal component of the mean velocity of the flow over the whole cross section of the trench. Figure 3b gives the field of the velocities in the trench. After the impact of the main jet, forming in the mixing zone, against the rear wall, a jet is propagated along the walls of the trench. Here a considerable part of the initial momentum of the jet is lost with its propagation along the rear wall. As with laminar flow, at the center of the jet there is a region with constant vorticity. The secondary eddy in the right-hand lower corner and the stagnant zone in the left-hand lower corner occupy an area which is several times smaller in comparison with laminar flow. In the corners of the trench and in the central part of the zone with constant vorticity, there were large fluctuations of the vector of the velocity, which considerably increased the error of the measurements in these regions.

Mixing Zone. Figure 4b ($1 - \bar{x} = 0.25$; $2 - \bar{x} = 0.35$; $3 - \bar{x} = 0.5$; $4 - \bar{x} = 0.65$; $5 - \bar{x} = 0.75$; $6 - \bar{x} = 0.95$; $7 - \bar{n} = 1/7$) gives the distribution of the velocity in the mixing zone at different distances from the point of breakaway of the flow. The velocity at the axis of the jet and at its outer boundary varies only slightly along the longitudinal coordinate. The relative value of the velocity at the outer boundary of the jet is considerably greater than the corresponding velocity with laminar flow. Here, for purposes of comparison, there is plotted also the exponential profile of the velocity in the channel ahead of the trench. It can be seen that the profile of the velocity in the jet, in comparison with laminar jet flow, is propagated considerably more rapidly. In Fig. 5c ($1 - \Pi = 0$; $2 - \Pi = -2$; $3 - \Pi = -5$; $4 - \Pi = -12$; $5 - \bar{x} = 0.25$; $6 - \bar{x} = 0.5$; $7 - \bar{x} = 0.75$) these same velocities are represented in the dimensionless coordinates

$$\bar{u} = (u - u_2)/(u_1 - u_2);\ \eta = y/\delta_{1/2}.$$

In such an analysis, the experimental profiles coincide with the theoretical profiles of the velocity in the transitional section of a turbulent jet [11] with different values of the parameter

$$\Pi = (\partial^2 \bar{u}/\partial \eta^2)_{\eta=0},$$

characterizing the gradient of the velocity at the axis of the jet, where

$$\bar{u} = (u-u_2)/(u_1-u_2);\ \eta = y/\delta;\ \bar{u} = (1-60f_2) + \Pi(f_1 - 12f_2);\ f_1 = (1/2)\eta^2 - (4/5)\eta^5 + (1/2)\eta^6;$$
$$f_2 = (1/4)\eta^4 - (2/5)\eta^5 + (1/6)\eta^6.$$

At the start of the transitional section of the jet the derivative of the velocity with respect to the longitudinal coordinate is equal to zero. Out of this flows the condition, characterizing the initial profile of the velocity, $\Pi = 0$. With large negative values of $\Pi(|\Pi| > 0)$, formula (3) gives a negative value of the velocity near the outer boundary of the jet, which is physically impossible. Consequently, for the transitional section of the jet the condition is satisfied:

$$-10 < \Pi < 0.$$

The Boundary Layer. With turbulent flow conditions in the channel ahead of the trench, the thickness of the boundary layer at the walls of the trench is small. Therefore, it is impossible to make measurements of the change in the velocity in the boundary layer. Some qualitative conclusions can be drawn with respect to the structure of the boundary layer by analyzing the change in the velocity at the limit of the boundary layer and in its thickness along the walls of the trench (see Fig. 6b). The character of the change in the velocity at the limit of the boundary layer is approximately the same as with laminar flow (see Fig. 6a). However, the greatest value of the dimensionless velocity reaches 0.4 at the rear wall, 0.27 at the bottom, and 0.23 at the front wall of the trench, which is higher than the corresponding values for Re = 1500. These values are approximately equal to the values of the maximal velocities in a boundary layer obtained in [7]. Breakaway of the boundary layer from the wall (see Fig. 6d) takes place with values of the Pohlhausen shape parameter Λ exceeding by an order of magnitude the breakaway value for a laminar boundary layer. This obviously constitutes evidence that the boundary layer in the present case is turbulent.

Outer Part of Jet near the Wall. In Fig. 5d ($1 - \bar{x} = 0.20$; $2 - \bar{x} = 0.35$; $3 - \bar{x} = 0.50$; $4 - \bar{x} = 0.65$; $5 - \bar{x} = 0.95$; $\bar{x}$ is reckoned along the line of maximal velocities) there are given experimental profiles of the velocity, worked up using the same method as in the external part of a laminar jet near the wall. For purposes of comparison, the same figure gives the profile of the velocities in the mixing zone (curve 7) at a distance $\bar{x} = 0.7$ (only the outer part of the velocity profile located below the line $y = 0$ is considered). Curve 6 is the universal profile of the velocity in accordance with formula (2.1). This comparison shows that the profiles of the velocity in the outer part of a turbulent jet retain the form of the initial distribution of the velocity in this part of the jet, which then turns in the trench and propagates along its walls. The kinematic momentum of this part of the jet in the mixing zone (below the line $y = 0$) is 20% greater than the momentum of the jet near the wall immediately after the rotation of the jet in the trench ($\bar{y} = 0.2$). The change in the maximal velocity in the jet near the wall along the line of the flow in the trench practically coincides with a curve calculated using the equation [16] $u_{max} = 2.40\sqrt{K/x}$. The initial value of the longitudinal coordinate x_0 was determined in the same way as with Re = 1500.

Core. The distribution of the vortical stress ω, parallel to the bottom of the trench, is shown in Fig. 8b [1) $\bar{v}$; 2) ω], where the section with constant vorticity has a greater extension in comparison with the corresponding section with laminar flow. An increase in the vorticity in the jet near the wall is observed only at the rear wall of the trench, and this increase is less than with Re = 1500.

In the experiments there was observed a certain lack of agreement in the mass flow rates for two symmetrical halves of the cross section of the trench; for example, for a horizontal section with y/H = 0.5 (Re = 1500), 12% more liquid flows in than flows out. With Re = 15,000, this lack of agreement is less, but it is still present. This phenomenon is, of course, not connected with the accuracy of the measurements of the velocity, which must enable drawing up a balance with an error not greater than 1-2%.

A possible explanation of this effect may be connected with the appearance of a certain lack of symmetry in the plane of the measurements, brought about by the friction of the central eddy at the end walls of the trench.

In [17] an approximate evaluation is made of the systematic and random errors arising in the measurement of the components of the vector of the mean velocity. The systematic errors due to inaccuracy in adjustment of the optical scheme of the laser Doppler velocimeter can be reduced to fractions of a percent. The errors in determination of the scale coefficient of the optical scheme are on the same order. When the experimental results are made dimensionless, errors of a systematic character are excluded. The most considerable contribution is that of errors connected with the character of the Doppler signal itself and with the procedure for its analysis by the electronic scheme of the laser Doppler velocimeter.

It is well known that even in the absence of turbulent pulsations and of a gradient of the velocity, within the limits of the measuring volume of the laser Doppler velocimeter, the spectrum of the Doppler signal has a finite width, inversely proportional to the flight time of the scattered particles through the measuring volume. This broadening of the spectrum, which in the literature has received the name of Doppler indeterminacy, can be evaluated using the formulas given in [18]. The variance appears in the form of a variation of the readings of the digital frequency meter of the laser Doppler velocimeter and in the form of noise at the outlet of the frequency detector, whose level bounds the lower limit of the measurement of the parameters of the turbulence. The error, calculated using the theoretical formula of [18], for Re = 1500 and an averaging time of 1 sec, is found equal to 0.1%.

The results of the experiments were used to evaluate the relative mean-square errors in determination of the mean velocity under laminar and turbulent flow conditions in different zones of the channel and the trench (the error was calculated as the variation coefficient over 10 readings of the digital frequency meter, with an averaging time of 1 sec). At the center of the channel with Re = 1500 the value of the error was 0.2% and at the wall of the channel, 1.2%. The rise in the variance of the readings in the zone near the wall is explained by the additional broadening of the spectrum of the Doppler signal due to the gradient of the velocity. Under turbulent conditions, the variance of the readings is also greater. The calculated values of the errors at the center of the channel and at the wall under turbulent conditions (Re = 15,000) were 0.5 and 2.5%, respectively. In the trench near the walls and at the bottom the error in measurement of the vector of the mean velocity under laminar conditions was within the limits of 2-3%, while, under turbulent conditions, as a result of the large fluctuations of the vector of the mean velocity, the variance of the readings reached 10%. At the corners of the trench and the central part of the core of the eddy, where the components of the vector of the velocity were variable in sign, measurements were not made. It is also necessary to take into consideration the shift in the evaluation of the mean velocity introduced by the electronic scheme of the filtration of the Doppler signal. Preliminary calculations showed that if the mean value of the frequency of the spectrum of the Doppler signal (for a laminar gradientless flow) does not coincide with the central frequency of the filter, then the maximal shift in the evaluation of the mean velocity is 1.5%.

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